This presentation provides an example of initial training in a high-leverage math research-based instructional practice that is aligned to campus instructional materials—Productive Struggle. Implementation support look-fors used for ongoing support in this practice are shared on slide 22.

Math RBIS 4: Productive Struggle

What are the essential best practices in mathematics instruction?

Math Research-based Instructional Strategies (RBIS)

1 Balance Conceptual & Procedural	2 Depth of Key Concepts	3 Coherence of Key Concepts	4 Productive Struggle	5 Assessment Practices
Pursue **rigor by balancing conceptual understanding, procedural skill and fluency**, and **application** as required by the standards in the TEKS.	Focus on math content that **aligns to and meets the rigor of the TEKS** for each grade level, **while concentrating time and effort** on going deep on the **most important topics** for the grade level.	**Connect concepts within and across grades** along a strategic progression of learning so that new understandings are built on previous foundations. Mathematics tells a **continuous, connected story**.	Students engage in productive problem solving, engaging in **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.	Leverage HQIM **embedded assessments** to drive instruction.

Warm Up

"Natalie has 30 jellybeans. Her mom gives her 23 more jellybeans. How many jellybeans does Natalie have now?"

- How might a student **struggle** to solve this problem?

2(4) (B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations

More information for you

This is an "add to result unknown" **problem type**.

It is the **simplest** problem type in terms of solving.

This type of problem is found in **Eureka Math TEKS Edition** in K-2nd grade. It becomes one of the two steps in some 3rd-5th grade multi-step problems.

Defining productive struggle

4

Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

"…students **expend effort to make sense of mathematics**, to figure something out that is not immediately apparent…The struggle we have in mind comes from **solving problems that are within reach and grappling with key mathematical ideas** that are comprehendible but not yet well formed (Hiebert et al., 1996)."

"…productive struggle comprises **the work that students do to make sense of a situation and determine a course of action when a solution strategy is not stated**, implied, or immediately obvious. From an equity perspective, this implies that each and **every student must have the opportunity to struggle** with challenging mathematics and to receive support that encourages their persistence without removing the challenge." (NCTM publications 2007, 2017)

Defining productive struggle

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Defining productive struggle

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Productive Struggle

Student engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

Maintains Rigor	Provides students time to collaboratively problem solve using different representations and then asking them to explain their thinking
Sets up all students to Engage	Tasks should have multiple entry points so that students can use different solution paths to solve and make connections
Develops Independent Problem Solvers	Acknowledging when students' effort supports their thinking and mathematical understanding, thus developing their capacity to persevere in the face of challenging content

Observing productive struggle…

4

Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

- **Observe and Reflect:** Watch the following instructional video.

Is this an example of productive struggle? Using language from the RBIS, explain why or why not.

"Natalie has 30 jellybeans. Her mom gives her 23 more jellybeans. How many jellybeans does Natalie have?"

Video source: Children's Mathematics Second Edition; Carpenter, Fennema, Franke, Levi, and Empson 2015

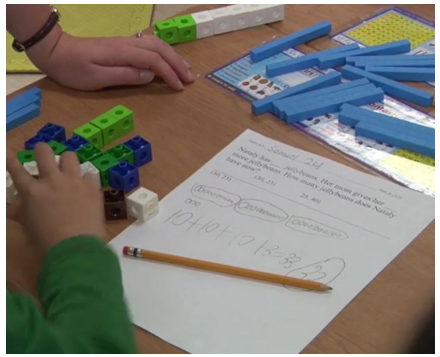
Observing productive struggle…

4

Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

- **Find a partner across the room:** Is this an example of productive struggle? Using language from the RBIS, explain why or why not.

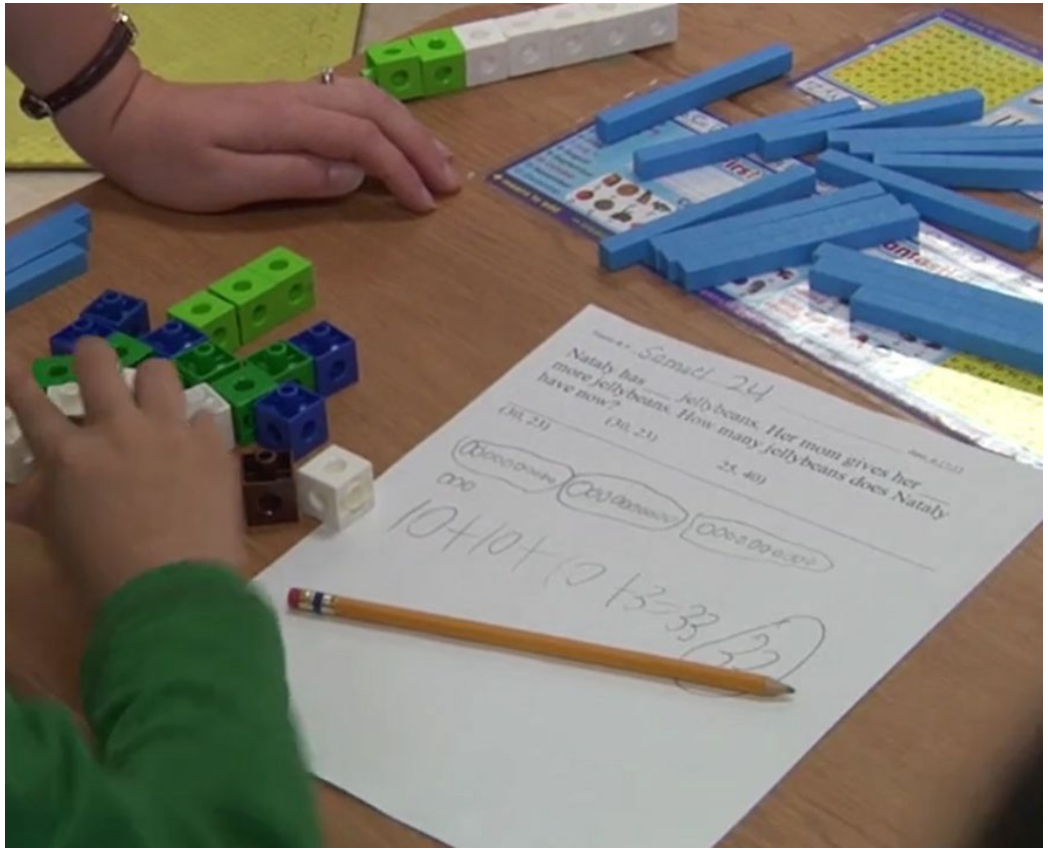


"Natalie has 30 jellybeans. Her mom gives her 23 more jellybeans. How many jellybeans does Natalie have?"

This was productive struggle because…

- The student had **multiple opportunities** to represent the two addends in the problem until his **representation** was correct. The student created the representations.

- The student **explained his thinking** and was able to **revise** his thinking based on **questioning and input** from the teacher rather than just repeated given steps.



Is this productive struggle?

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Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

- **Observe and Reflect:** Watch the following instructional video.

Is this an example of productive struggle? Using language from the RBIS, explain why or why not.

"Yesterday I went to the store and bought 8 boxes of candy. There are 23 pieces of candy in each box. How much candy did I buy?"

Video source: Children's Mathematics Second Edition; Carpenter, Fennema, Franke, Levi, and Empson 2015

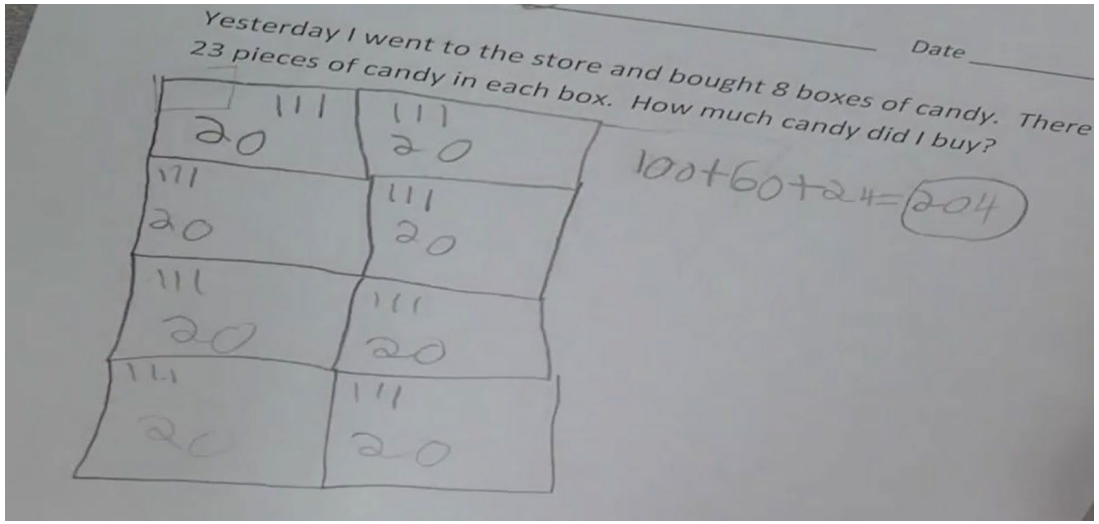
Is this productive struggle?

4

Productive Struggle

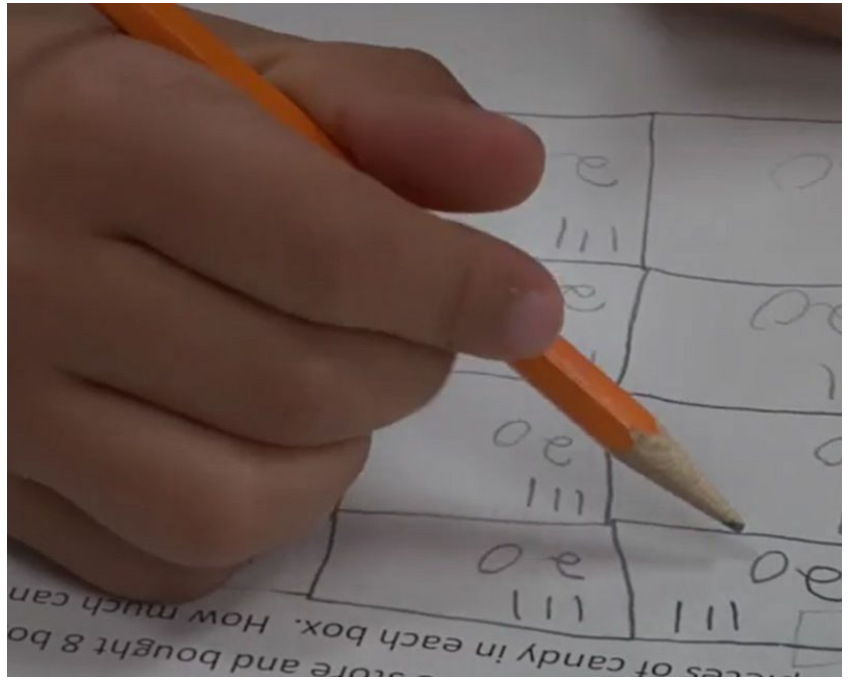
Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

- **Return to your partner:** Is this an example of productive struggle? Using language from the RBIS, explain why or why not.



"Yesterday I went to the store and bought 8 boxes of candy. There are 23 pieces of candy in each box. How much candy did I buy?"

This was productive struggle because…



- The student engaged in the problem using **representations** and was able to **explain** his thinking.
- The student was able to **revise** his thinking based on **input** from the teacher around using a different operation that allowed for more **efficient** problem solving.

Nonexamples of Productive Struggle

A

A student cannot get started.

Teacher: What have you figured out so far?
Student: Nothing. I am not sure what to do.
Teacher: The first thing you need to do is to figure out how much money she makes every hour. Do you remember what operation you need to use to do that?
Student: Divide?
Teacher: What are you going to divide?
Student: 2 ÷ 24?
Teacher: No, 24 ÷ 2.
Student: So, it's 12.
Teacher: So this is the amount she makes per hour. Now you need to divide 80 by 12.

B

A student is working and has the wrong answer.

Teacher: Uh oh! Your answer is wrong. Do you remember our regrouping poem?

Student: Yes! More on top? No need to stop.
Teacher: Right, but you don't have more on top, so you need to…
Student: Go next door and get more.

Teacher: Yes, you need to go to the tens place and regroup and add ten to the ones place. Now you can subtract.
Student: 12 minus 6 is 6.
Teacher: Great job, you got it!

Reflect: Using RBIS language, explain why these are nonexamples.

Productive Struggle is NOT

- students just showing their work on paper
- something to give only when students are at a certain age or grade level
- asking all students how they got an answer
- just for students who have the wrong answer.
- giving students challenging work or work above grade level.
- something that could harm a student's development

5 Types of Common Teacher Responses to Struggle

	Example	Impact on Student Learning
1. Telling 2. Directed Guidance	*"Not quite. Instead, draw 23 cubes…"*	More likely to lower the demands of the task and remove struggle.
3. Unfocused or Vague Direction	*"Read the problem again and check your work…"*	More likely to provide general suggestions that are not helpful to the particular task.
4. Probing Guidance 5. Affordance	*"Can you tell me how your picture shows how you solved the problem…"*	More likely to maintain the demands of the task and support productive struggle

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." Journal of Mathematics Teacher Education 17, no.4 (2015): 375-399. [p. 387]

Strategies to Support Productive Struggle

Strategy	What it looks like
Question	"Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem."
Encourage	"Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers."
Give Time	"Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students."
Acknowledge	"Teachers acknowledge that struggle is an important part of learning and doing mathematics."

Probing Guidance

Affordance

What examples of these have you observed today?

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." Journal of Mathematics Teacher Education 17, no.4 (2015): 375-399. [p. 387]

HQIM Task Analysis

- As we analyze tasks, look for opportunities where students could:
  - grapple with key mathematical ideas.
  - determine a course of action when a solution strategy is not named.

- As we analyze tasks, look for opportunities where teachers could:
  - maintain the demands of the task.
  - support productive struggle.

5 (3) (G) ...solve for quotients of decimals...using strategies and algorithms, including the standard algorithm

Eureka Math TEKS Edition, Grade 5, Module 2, Lesson 30

5th Grade Multi-Digit Decimal Division **Student Task**

1. 156 ÷ 24 and 102 ÷ 15 both have a quotient of 6 and a remainder of 12.
   a. Are the division expressions equivalent to each other? Use your knowledge of decimal division to justify your answer.

   b. Construct your own division problem with a two-digit divisor that has a quotient of 6 and a remainder of 12 but is not equivalent to the problems in 1(a).

5th Grade Multi-Digit Decimal Division **TE Guidance**

- Turn to a partner, compare your work and thinking for Problem 1(b). (Take the necessary time here for students to compare approaches. Possibly, give the students the following challenge: Is it possible to create a pair of division problems whose quotient and whole number remainder look equal and actually are equal when the decimal division is used?)

HQIM Task Analysis

- As we analyze tasks, look for opportunities where students could:
  - grapple with key mathematical ideas.
  - determine a course of action when a solution strategy is not named.

- As we analyze tasks, look for opportunities where teachers could:
  - maintain the demands of the task.
  - support productive struggle.

7 (4) (D)...solve problems involving ratios, rates and percent, including multi-step problems involving percent increase and percent decrease

Carnegie Learning Texas Math Solution, Module 2, Topic 1, Lesson 4, Activity 4.3

7th Grade Proportional Relationships **Student Task**

3. Elinor wanted to order an 18-inch hoagie, which costs $7.99. The sandwich shop is out of 18-inch buns. They only have 12-inch buns.
a. What should be the percent decrease in the cost of the hoagie?
b. What should the new cost be?

7th Grade Proportional Relationships **TE Guidance**

- What formula is used to calculate the area of a pizza?
- How do you determine the percent decrease of the hoagie?
- Is the smaller hoagie $\frac{1}{2}$ the size of the larger hoagie?
- Is the smaller hoagie $\frac{3}{4}$ the size of the larger hoagie?
- Is the smaller hoagie $\frac{2}{3}$ the size of the larger hoagie?

Differentiation strategy

To extend the activity, have students investigate measures associated with the 2 cubes in more depth. Create a template similar to the one shown. Have students calculate the percent of increase between the two cubes for all of the measures (length of an edge, area of a face, surface area, and volume).

Dos and Don'ts of Productive Struggle

DON'T…	Rationale
Don't provide **support too quickly**.	*You will likely undermine your preparation and your students' abilities to persevere.*
Don't **offer** to get students started or **tell** them how to move to the next step.	*Students need to stretch their thinking to learn. When you provide too much information, you take away the struggle.*
Don't suggest only **one solution** path.	*Students may initially be unable to get started out of fear of not knowing the correct first step or the "right way" to solve a problem. Students are more likely to succeed if they choose a strategy that feels natural to them.*
Don't focus your **praise** on **correct** answers.	*Focus it on the behaviors and strategies that lead to understanding.*

The Struggle is Real (and Productive) Mike Linskey, Great Minds: Eureka Math Blog https://gm.greatminds.org/math/blog/eureka/the-struggle-is-real-and-productive

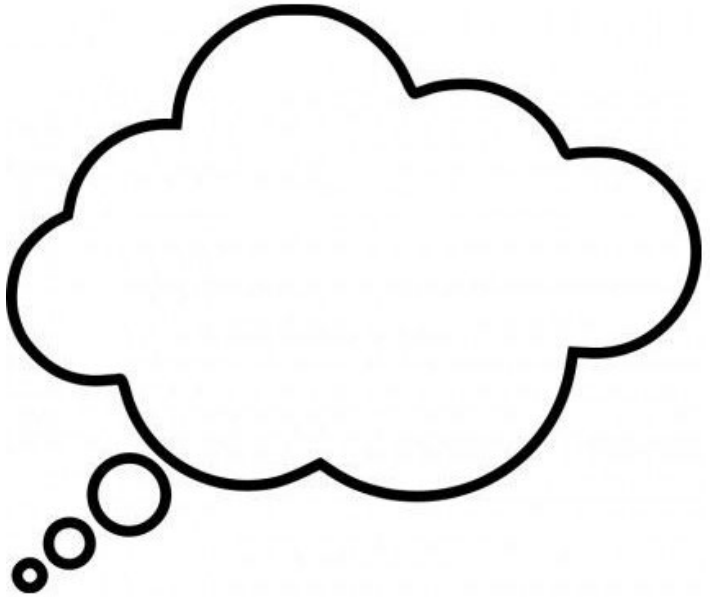
Dos and Don'ts of Productive Struggle

DO…	Rationale
Give your students **time** to engage in productive struggle.	*You know your students, and you prepared your lesson to meet their needs. Trust that they can accomplish what you prepared for them.*
Ask questions when students are stuck.	*What have you done before that might be useful now? What seems important in the problem? How is this the same or different as what you've seen before?*
Encourage students to solve problems in different ways.	*Students need to feel comfortable trying different strategies. Celebrate creativity by encouraging students to share their thinking with the class.*
Praise students' effort on both successful and unsuccessful **attempts**.	*These actions send the message to students that you value risk-taking and trying out ideas. Have students reflect on what they learned from their unsuccessful efforts and how those efforts helped them decide what method(s) to try next.* ***Math is not just about getting the right answer.***

The Struggle is Real (and Productive) Mike Linskey, Great Minds: Eureka Math Blog https://gm.greatminds.org/math/blog/eureka/the-struggle-is-real-and-productive

Table Talk
Closing Reflection

Why is productive struggle important to student learning? How does it connect to previous RBIS?



Productive Struggle Training Implementation

- Teachers will be supported in showing evidence of productive struggle implementation through learning walks and coaching.
- The following look-fors will serve as the anchor for implementation support.

Look-fors in classroom observations:

- Teacher provides students time to collaboratively problem solve using different representations and then asking them to explain their thinking.
- Teacher provides students time to engage in productive struggle and does not intervene too quickly.
- Teacher asks questions when students are stuck, utilizing Probing Guidance and Affordance rather than providing students the correct answer.
- Teacher encourages students to solve problems in multiple ways and does not suggest there is only one way to solve a problem.
- Teacher equally praises student effort on both successful and unsuccessful attempts – does not praise only correct answers.

Look-fors in materials:

- Lesson materials provide the intended coherence of the TEKS to provide practice opportunities to build student skill and confidence prior to engaging in tasks fully aligned to the on-grade level TEKS.
- Lesson materials and/or lesson plan provide opportunities for student discussion around the key concepts of the lesson.
- Lesson materials and/or lesson plan identify common, high-leverage errors or misconceptions students may have and pre-plan teacher moves as a solution pathway.